EME172
Homework #4 Solution

2. Problem 6-2:

s^5	1	4	3
s^4	-1	-4	-2
s^3	ε	1	0
s^2	$\frac{1-4\varepsilon}{\varepsilon}$	-2	0
s^1	$\frac{2\varepsilon^2+1-4\varepsilon}{1-4\varepsilon}$	0	0
s^0	-2	0	0

3 rhp, 2 lhp

3. Problem 6-12:

$$T(s)=\frac{1}{2s^4+5s^3+s^2+2s+1}$$

s^4	2	1	1
s^3	5	2	0
s^2	1	5	
s^1	-23	0	
s^0	5		

Total: 2 lhp, 2 rhp

4. Problem 6-25:

$$T(s)=\frac{K(s-2)(s+4)(s+5)}{Ks^3+(7K+1)s^2+2Ks+(3-40K)}$$

s^3	K	2K
s^1	$\frac{54K^2-K}{7K+1}$	0
s^0	3-40K	0

For stability, $\frac{1}{54}<K<\frac{3}{40}$

5. Problem 6-27:

$$T(s) = \frac{K}{s^3 + 80s^2 + 2001s + (K + 15390)}$$

s^3	1	2001
s^2	80	K+15390
s^1	$-\frac{1}{80}K + \frac{14469}{8}$	0
s^0	K+15390	0

There will be a row of zeros at s^1 row if $K = 144690$. The previous row, s^2, yields the auxiliary equation, $80s^2 + (144690 + 15390) = 0$. Thus, s = ± j44.73. Hence, $K = 144690$ yields an oscillation of 44.73 rad/s.

6. Problem 6-49:

$$T(s) = \frac{0.7K(s+0.1)}{s^4 + 2.2s^3 + 1.14s^2 + 0.193s + (0.07K+0.01)}$$

s^4	1	1.14	0.07K+0.01
s^3	2.2	0.193	0
s^2	1.0523	0.07K+0.01	0
s^1	0.17209 - 0.14635K	0	0
s^0	0.07K+0.01	0	0

For stability, $-0.1429 < K < 1.1759$

7. Problem 7-9:

a. The closed-loop transfer function is,

$$T(s) = \frac{5000}{s^2 + 75s + 5000}$$

from which, $\omega_n = \sqrt{5000}$ and $2\zeta\omega_n = 75$. Thus, $\zeta = 0.53$ and

$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} x100 = 14.01\%$.

b. $T_s = \frac{4}{\zeta\omega_n} = \frac{4}{75/2} = 0.107$ second.

c. Since system is Type 1, e_{ss} for $5u(t)$ is zero.

d. Since K_v is $\frac{5000}{75} = 66.67$, $e_{ss} = \frac{5}{K_v} = 0.075$.

e. $e_{ss} = \infty$, since system is Type 1.

8. Problem 7-24:

The system is stable for $0 < K < 2000$. Since the maximum K_v is $K_v = \frac{K}{320} = \frac{2000}{320} = 6.25$, the minimum steady-state error is $\frac{1}{K_v} = \frac{1}{6.25} = 0.16$.

9. Problem 7-25:

To meet steady-state error characteristics:

$$E(s) = \frac{R(s)}{1 + G(s)} = \frac{1}{s\left(1 + \frac{K(s+\alpha)}{(s+\beta)^2}\right)}$$

$$e(t)\Big|_{t\to\infty} = sE(s)\Big|_{s\to 0} = \frac{1}{1 + \frac{K\alpha}{\beta^2}} = \frac{\beta^2}{\beta^2 + K\alpha} = 0.1$$

Therefore, $K\alpha = 9\beta^2$.

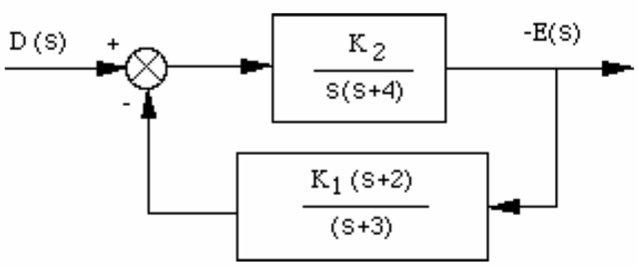
To meet the transient requirement: Since $T(s) = \frac{K(s+\alpha)}{s^2 + (K+2\beta)s + (\beta^2 + K\alpha)}$,

$\omega_n^2 = 10 = \beta^2 + K\alpha$; $2\zeta\omega_n = \sqrt{10} = K+2\beta$. Solving for β, $\beta = \pm 1$. For $\beta = +1$, $K = 1.16$ and $\alpha = 7.76$.

An alternate solution is $\beta = -1$, $K = 5.16$, and $\alpha = 1.74$.

10. Problem 7-35:

Error due only to disturbance: Rearranging the block diagram to show D(s) as the input,



Therefore,

$$-E(s) = D(s)\frac{\frac{K_2}{s(s+4)}}{1 + \frac{K_1K_2(s+2)}{s(s+3)(s+4)}} = D(s)\frac{K_2(s+3)}{s(s+3)(s+4) + K_1K_2(s+2)}$$

For $D(s) = \frac{1}{s}$, $e_D(\infty) = \lim_{s\varnothing 0} sE(s) = -\frac{3}{2K_1}$.

Error due only to input: $e_R(\infty) = \frac{1}{K_v} = \frac{1}{\frac{K_1K_2}{6}} = \frac{6}{K_1K_2}$.

Design:

$e_D(\infty) = -0.000012 = -\frac{3}{2K_1}$, or $K_1 = 125{,}000$.

$e_R(\infty) = 0.003 = \frac{6}{K_1K_2}$, or $K_2 = 0.016$