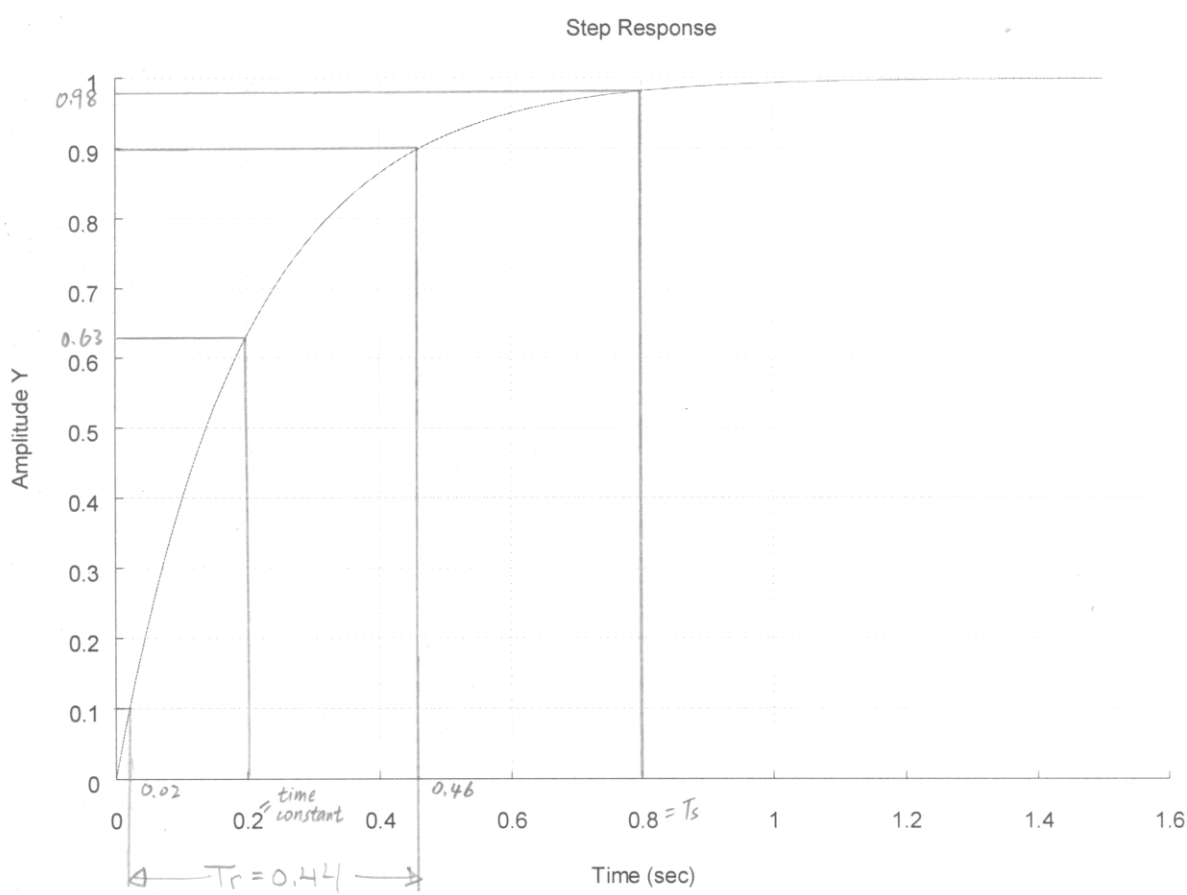
EME172
Homework #3 Solution

2. Problem 4-2 (a):

Program:

```
/* p4-2a.ch */
#include <control.h>

int main() {
    array double num[1] = {5},
                 den[2] = {1, 5};
    CControl sys;
    CPlot plot;

    sys.model("tf", num, den);
    sys.grid(1);
    sys.step(&plot, NULL, NULL, NULL);

    return 0;
}
```

Output:



3. Problem 4-18

b. $s^2+9s+18$, $\omega_n^2 = 18$, $2\zeta\omega_n = 9$, Therefore $\zeta = 1.06$, $\omega_n = 4.24$, overdamped.

d. $s^2+6s+144$, $\omega_n^2 = 144$, $2\zeta\omega_n = 6$, Therefore $\zeta = 0.25$, $\omega_n = 12$, underdamped.

e. s^2+9, $\omega_n^2 = 9$, $2\zeta\omega_n = 0$, Therefore $\zeta = 0$, $\omega_n = 3$, undamped.

f. $s^2+20s+100$, $\omega_n^2 = 100$, $2\zeta\omega_n = 20$, Therefore $\zeta = 1$, $\omega_n = 10$, critically damped.

4. Problem 4-20 (a, b):

a. $\omega_n^2 = 16$ r/s, $2\zeta\omega_n = 3$. Therefore $\zeta = 0.375$, $\omega_n = 4$. $T_s = \frac{4}{\zeta\omega_n} = 2.667$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$ 0.8472 s; $\%OS = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100 = 28.06$ %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.4238$; therefore, $T_r = 0.356$ s.

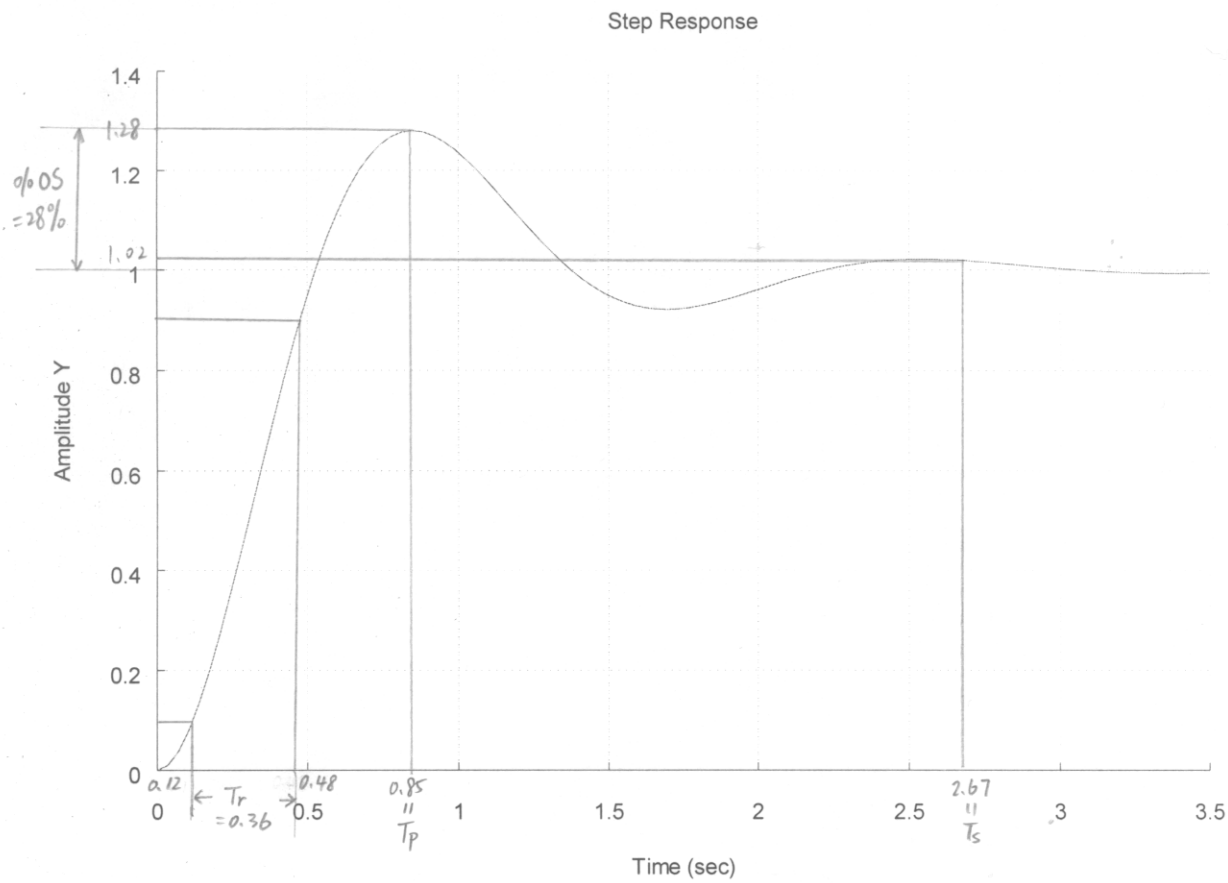
b. $\omega_n^2 = 0.04$ r/s, $2\zeta\omega_n = 0.02$. Therefore $\zeta = 0.05$, $\omega_n = 0.2$. $T_s = \frac{4}{\zeta\omega_n} = 400$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$ 15.73 s; $\%OS = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100 = 85.45$ %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 5.26$ s.

5. Problem 4-20 (a, b):

Program:

```
/* p4-20a.ch */
#include <control.h>

int main() {
    array double num[1] = {16},
                 den[3] = {1, 3, 16};
    CControl sys;
    CPlot plot;

    sys.model("tf", num, den);
    sys.grid(1);
    sys.step(&plot, NULL, NULL, NULL);

    return 0;
}
```

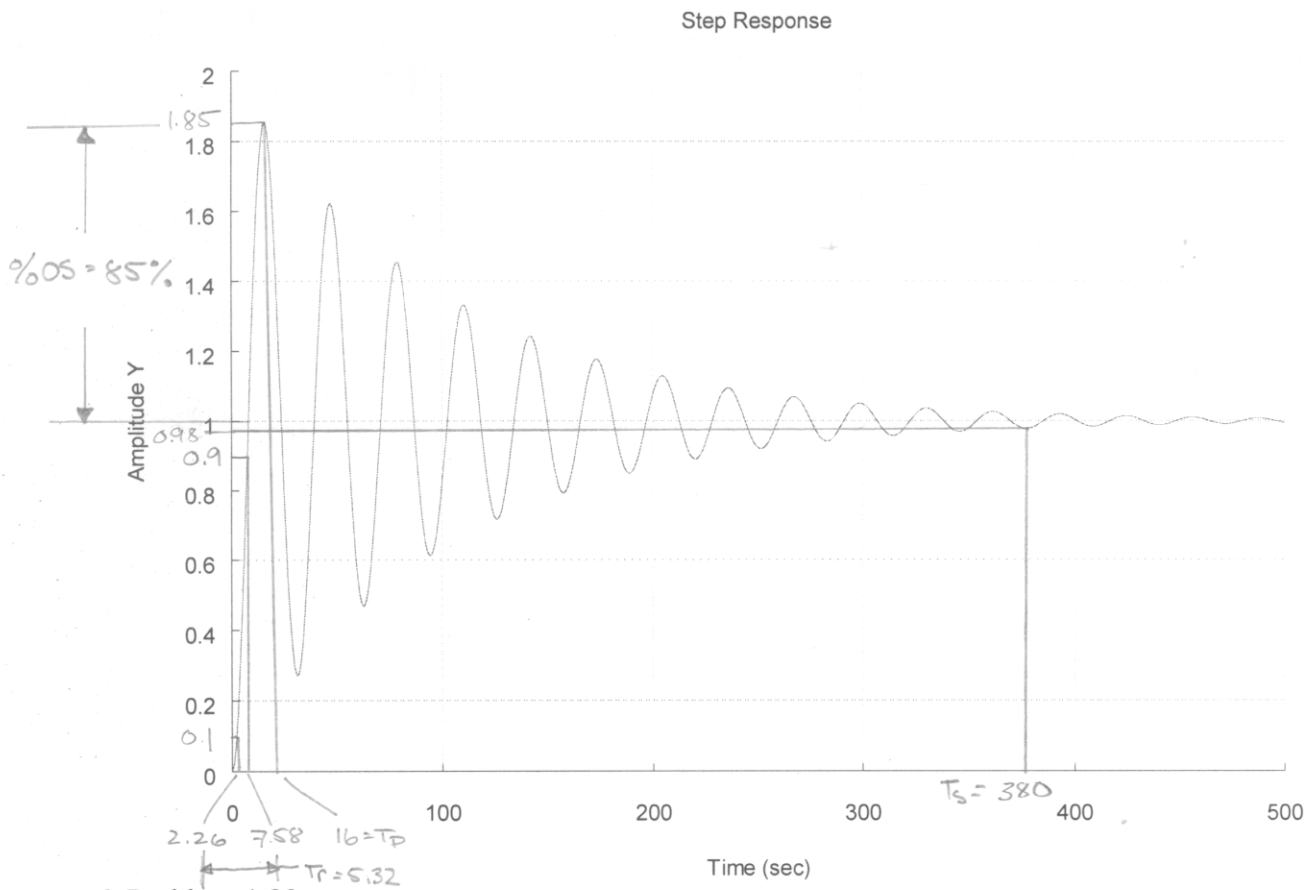
Output:



Program:

```
/* p4-20b.ch */
#include <control.h>

int main() {
    array double num[1] = {0.04},
                 den[3] = {1, 0.02, 0.04},
                 yout[500],
                 tout[500];
    double tf = 500;
    CControl sys;
    CPlot plot;

    sys.model("tf", num, den);
    sys.grid(1);
    sys.step(&plot, yout, tout, NULL, tf);

    return 0;
}
```

Output:



6. Problem 4-23

a. $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.56$, $\omega_n = \dfrac{4}{\zeta T_s} = 11.92$. Therefore, poles $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$= -6.67 \pm j9.88$.

b. $\zeta = \dfrac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}} = 0.591$, $\omega_n = \dfrac{\pi}{T_P\sqrt{1-\zeta^2}} = 0.779$.

Therefore, poles $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.4605 \pm j0.6283$.

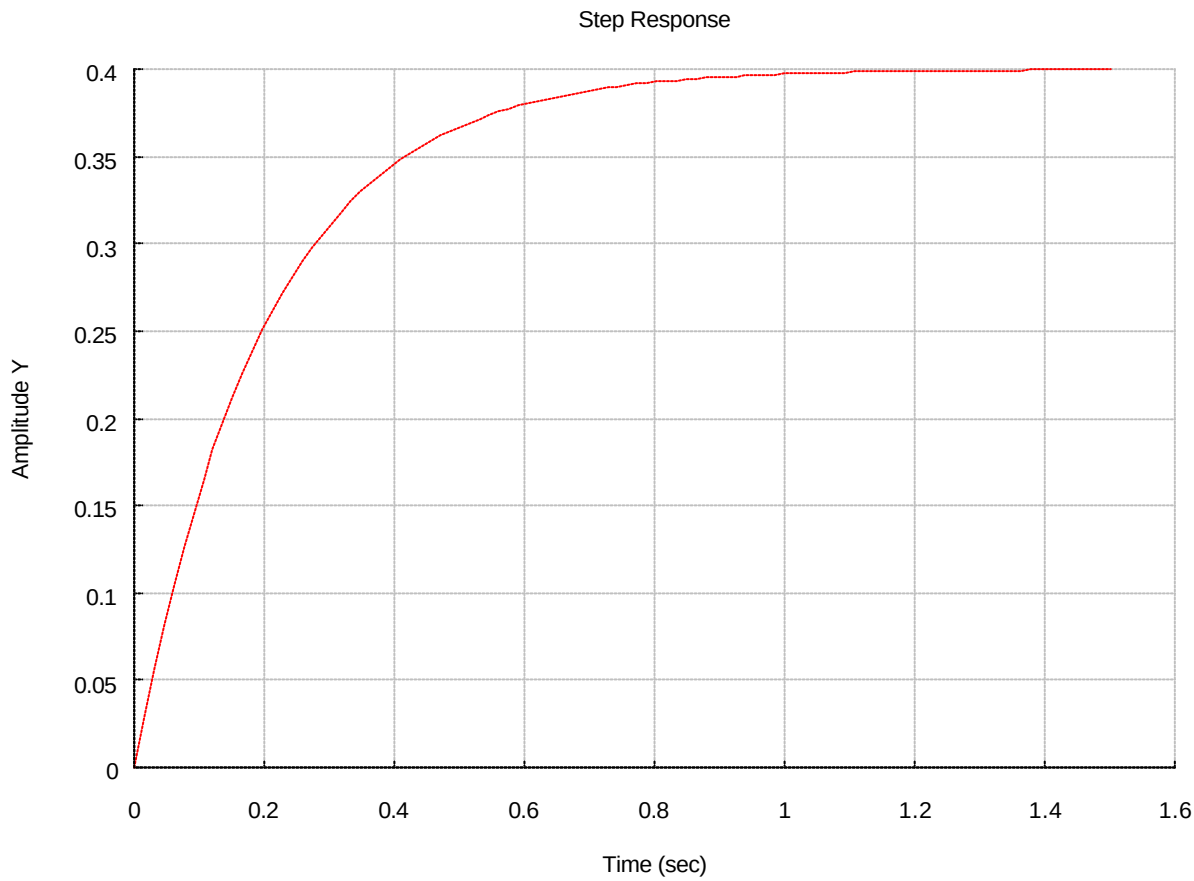
c. $\zeta\omega_n = \dfrac{4}{T_s} = 0.571$, $\omega_n\sqrt{1-\zeta^2} = \dfrac{\pi}{T_p} = 1.047$. Therefore, poles $= -0.571 \pm j1.047$.

7. Problem 4-40:

Program:

```
/* p4-40.ch */
#include <control.h>

int main() {
    array double A[3][3] = {-3,  1,  0,
                             0, -6,  1,
                             0,  0, -5},
                 B[3][1] = {0,
                            1,
                            1},
                 C[3] = {0, 1, 1},
                 D[1] = {0};

    CControl sys;
    CPlot plot;

    sys.model("ss", A, B, C, D);
    sys.grid(1);
    sys.step(&plot, NULL, NULL, NULL);

    return 0;
}
```

Output:



8. Problem 4-50 (a, c):

a. $\omega_n = \sqrt{10} = 3.16$; $2\zeta\omega_n = 4$. Therefore $\zeta = 0.632$. $\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} * 100 = 7.69\%$.

$T_s = \dfrac{4}{\zeta\omega_n} = 2$ seconds. $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.28$ seconds. From Figure 4.16, $T_r\omega_n = 1.93$.

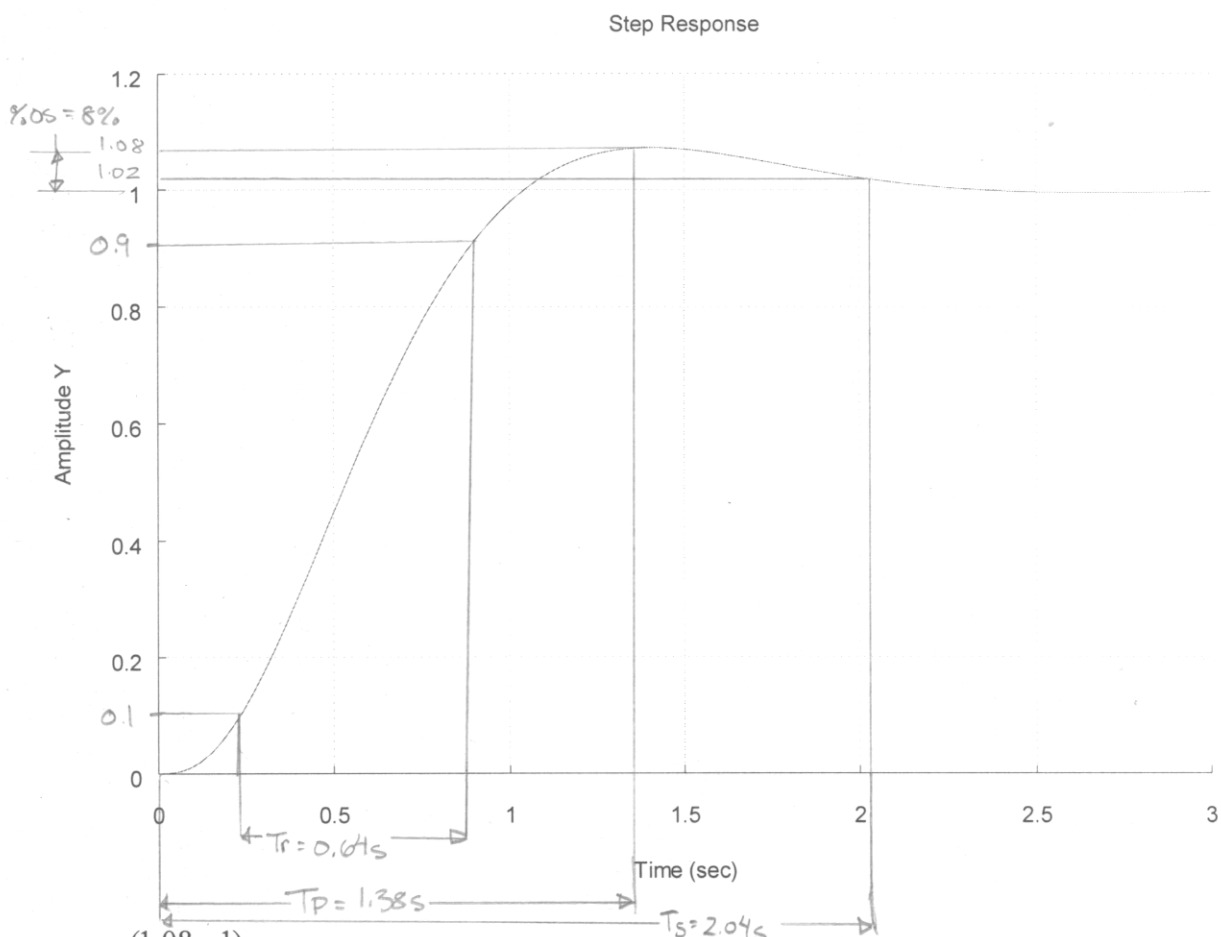
Thus, $T_r = 0.611$ second. To justify second-order assumption, we see that the dominant poles are at –

$2 \pm j2.449$. The third pole is at -10, or 5 times further. The second-order approximation is valid.

Program:

```
/* p4-50a.ch */
#include <control.h>

int main() {
    array double num[1] = {100},
                 den1[2] = {1, 10},
                 den2[3] = {1, 4, 10},
                 den[4];
    CControl sys;
    CPlot plot;

    conv(den, den1, den2);
    sys.model("tf", num, den);
    sys.grid(1);
    sys.step(&plot, NULL, NULL, NULL);

    return 0;
}
```

Output:



$$\%OS = \frac{(1.08-1)}{1} \times 100 = 8\%$$

9. Problem 4-56:

Writing the equation of motion: $(Js^2+s+K)\theta(s) = T(s)$. Therefore the transfer function is

$$\frac{\theta(s)}{T(s)} = \frac{\frac{1}{J}}{s^2+\frac{1}{J}s+\frac{K}{J}}.$$

$$\zeta = \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\pi^2+\ln^2\left(\frac{\%OS}{100}\right)}} = 0.358.$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{\frac{1}{2J}} = 8J = 4.$$

Therefore $J = \frac{1}{2}$. Also, $T_s = 4 = \frac{4}{\zeta\omega_n} = \frac{4}{(0.358)\omega_n}$. Hence, $\omega_n = 2.793$. Now, $\frac{K}{J} = \omega_n^2 = 7.803$. Finally, $K = 3.901$.

10. Problem 5-14:

Since $G(s) = \frac{K}{s(s+30)}$, $T(s) = \frac{G(s)}{1+G(s)} = \frac{K}{s^2+30s+K}$. Therefore, $2\zeta\omega_n = 30$. Thus, $\zeta = 15/\omega_n = 0.456$ (i.e. 20% overshoot). Hence, $\omega_n = 32.89 = \sqrt{K}$. Therefore $K = 1082$.