EME172
Homework #1 Solution

2.
$$\ddot{y} + 5\dot{y} + 4y = 3 \quad y(0) = 1 \quad \dot{y}(0) = 0$$

Taking the Laplace transform of the above ODE:

$$\left[s^2Y(s) - sy(0) - \dot{y}(0)\right] + 5\left[sY(s) - y(0)\right] + 4Y(s) = \frac{3}{s}$$
$$\left[s^2Y(s) - s\right] + 5\left[sY(s) - 1\right] + 4Y(s) = \frac{3}{s}$$
$$(s^2 + 5s + 4)Y(s) = s + 5 + \frac{3}{s} = \frac{s^2 + 5s + 3}{s}$$
$$Y(s) = \frac{s^2 + 5s + 3}{s(s^2 + 5s + 4)} = \frac{s^2 + 5s + 3}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4} \quad \text{where } A = \frac{3}{4}, B = \frac{1}{3}, C = -\frac{1}{12}$$

Taking the inverse Laplace transform of Y(s) to obtain y(t):

$$y(t) = \frac{3}{4} + \frac{1}{3}e^{-t} - \frac{1}{12}e^{-4t}$$

3.
$$\ddot{x} + 8\dot{x} + 25x = 10u(t) \quad x(0) = 0 \quad \dot{x}(0) = 0$$

Taking the Laplace transform of the above ODE:

$$\left[s^2X(s) - sx(0) - \dot{x}(0)\right] + 8\left[sX(s) - x(0)\right] + 25Y(s) = \frac{10}{s}$$
$$s^2X(s) + 8sX(s) + 25X(s) = \frac{10}{s}$$
$$X(s) = \frac{10}{s(s^2 + 8s + 25)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 8s + 25} \quad \text{where } A = \frac{2}{5}, B = -\frac{2}{5}, C = -\frac{16}{5}$$
$$X(s) = \frac{2}{5}\frac{1}{s} - \frac{2}{5}\frac{s+4}{(s+4)^2 + 3^2} - \frac{8}{15}\frac{3}{(s+4)^2 + 3^2}$$

Taking the inverse Laplace transform of X(s) to obtain x(t):

$$x(t) = \frac{2}{5} - \frac{2}{5}e^{-4t}\cos(3t) - \frac{8}{15}e^{-4t}\sin(3t)$$

4. Problem 2-7

The Laplace transform of the differential equation, assuming zero initial conditions, is,

$(s^3+3s^2+5s+1)Y(s) = (s^3+4s^2+6s+8)X(s)$.

Solving for the transfer function, $\dfrac{Y(s)}{X(s)} = \dfrac{s^3+4s^2+6s+8}{s^3+3s^2+5s+1}$.

5. Problem 2-8

a. Cross multiplying, $(s^2+2s+7)X(s) = F(s)$.

Taking the inverse Laplace transform, $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 7x = f(t)$.

b. Cross multiplying after expanding the denominator, $(s^2+15s+56)X(s) = 10F(s)$.

Taking the inverse Laplace transform, $\dfrac{d^2x}{dt^2} + 15\dfrac{dx}{dt} + 56x = 10f(t)$.

c. Cross multiplying, $(s^3+8s^2+9s+15)X(s) = (s+2)F(s)$.

Taking the inverse Laplace transform, $\dfrac{d^3x}{dt^3} + 8\dfrac{d^2x}{dt^2} + 9\dfrac{dx}{dt} + 15x = \dfrac{df(t)}{dt} + 2f(t)$.

6. Problem 2-26

$$\left(s^2+3s+2\right)X_1(s) - (s+1)X_2(s) = 0$$

$$-(s+1)X_1(s) + \left(s^2+2s+1\right)X_2(s) = F(s)$$

Solving for $X_1(s)$; $X_1 = \dfrac{\left|\begin{pmatrix} 0 & -[s+1] \\ F & s^2+2s+1 \end{pmatrix}\right|}{\left|\begin{pmatrix} s^2+3s+2 & -[s+1] \\ -[s+1] & s^2+2s+1 \end{pmatrix}\right|} = \dfrac{F(s)}{s^3+4s^2+4s+1}$. Thus, $\dfrac{X_1}{F(s)} = \dfrac{1}{s^3+4s^2+4s+1}$

7. Problem 2-42

$$\frac{K_t}{R_a} = \frac{T_{stall}}{E_a} = \frac{100}{50} = 2\ ;\ K_b = \frac{E_a}{\omega_{no-load}} = \frac{50}{150} = \frac{1}{3}$$

Also,

$$J_m = 2+18\left(\frac{1}{3}\right)^2 = 4;\ D_m = 2+36\left(\frac{1}{3}\right)^2 = 6.$$

Thus,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{2/4}{s(s+\frac{1}{4}(6+\frac{2}{3}))} = \frac{1/2}{s(s+\frac{5}{3})}$$

Since $\theta_L(s) = \frac{1}{3}\,\theta_m(s)$,

$$\frac{\theta_L(s)}{E_a(s)} = \frac{\frac{1}{6}}{s(s+\frac{5}{3})}.$$